KUKA KR500 R2830 KINEMATICS & MOTION PLANNING

MAX HEIL
ME7751, DR. HAIJUN SU

AGENDA

INTRODUCTION & ROBOT SELECTION

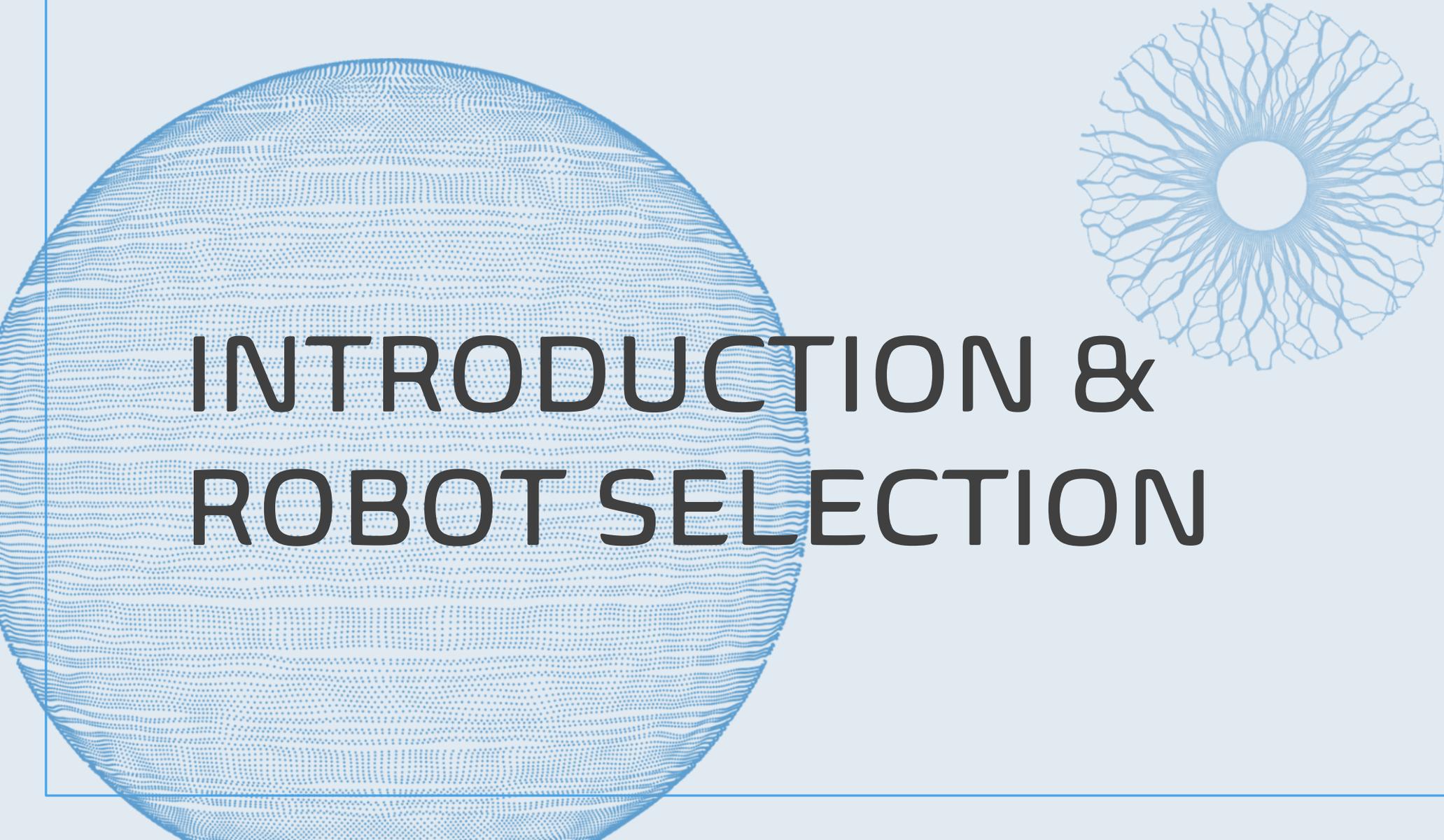# RIDE VEHICLE ROBOTICS

I have long been interested in exploring robotics for amusement applications. The ride vehicles are often complex, but very versatile.

- This is an example from a ride at Universal Orlando

- Uses a KUKA robot to move the end-effector (riders)

- Attached to a mobile robot unit on track

KUKA KR500 R2830

Simplify the robot system. For the purpose of this project, I needed to simplify the robotic system to just the robotic arm. Assume it is a stationary/fixed base.
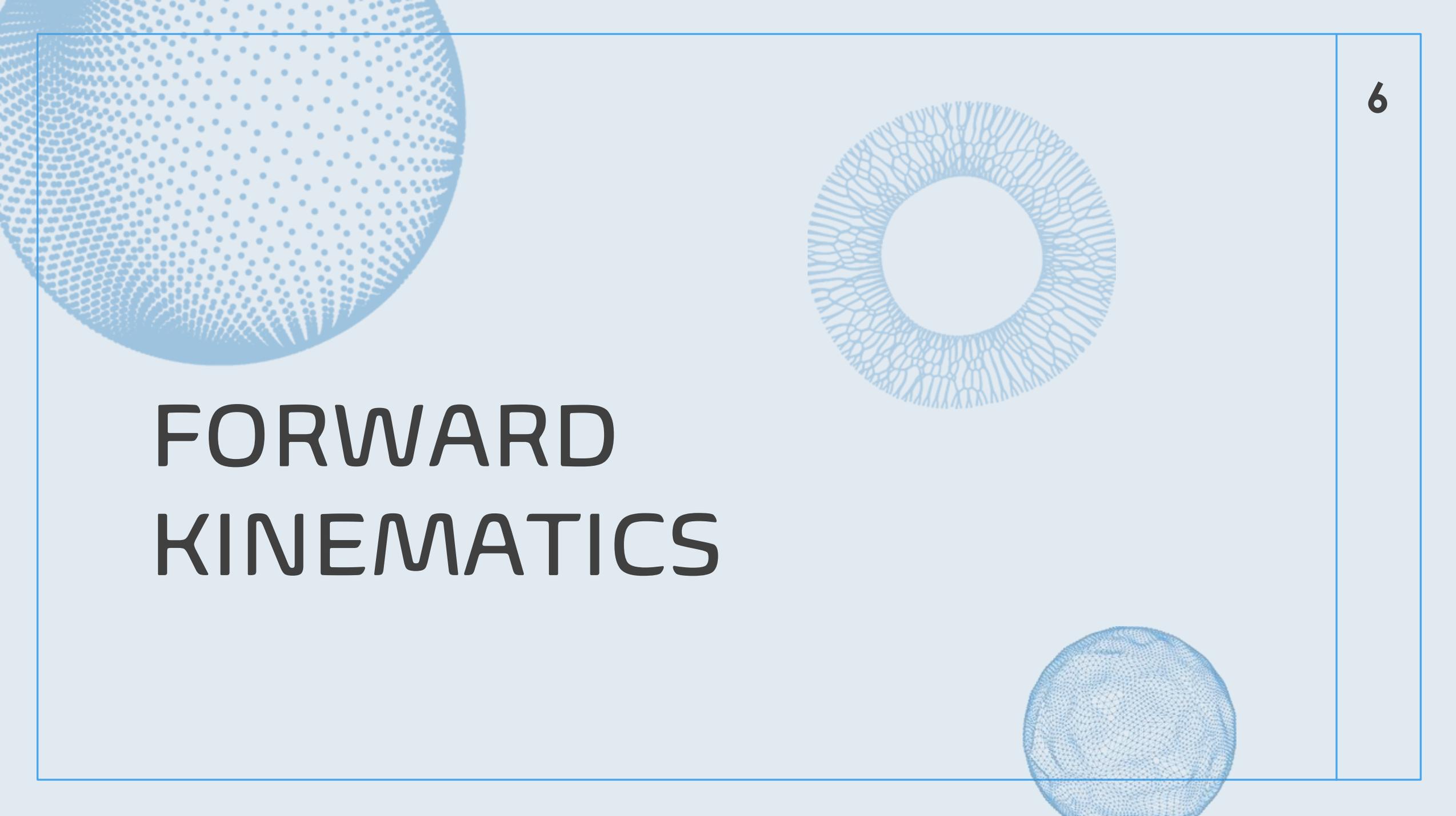
Select a robot close enough to the actual thing. KUKA makes a special brand of robotic arms (RoboCoaster KR500 Series), which do not have readily available information online.

Ensure spherical wrist for IK analysis. Make IK analysis easier by using a robot with spherical wrist.
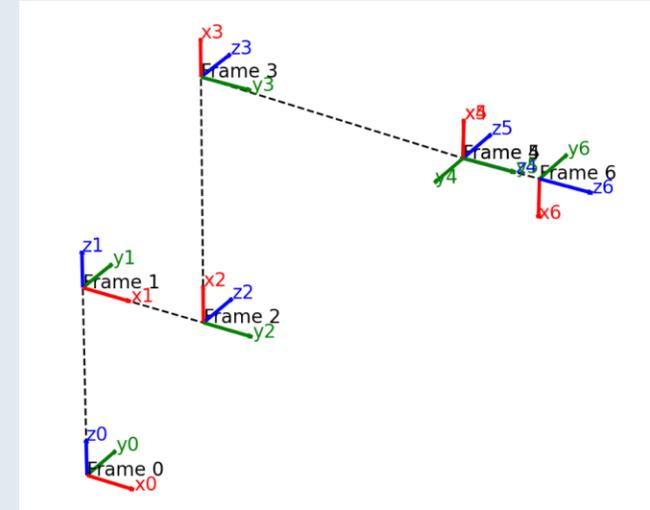
α (rad)	a (mm)	d (mm)	θ_offset (rad)
0	0	1045	0
-pi/2	500	0	0
0	1300	0	-pi/2
-pi/2	-55	1025	0
+pi/2	0	0	0
-pi/2	0	290	+pi

MODIFIED DH TABLE

FORWARD KINEMATICS

FK DERIVATION OVERVIEW

The six transforms with the KR500 parameters substituted:

The modified DH transformation from frame (i-1) to frame i is:

$$i-1T_i = \begin{bmatrix} c_i & -s_i & 0 & a_i \\ s_i c_{\alpha i} & c_i c_{\alpha i} & -s_{\alpha i} & -s_{\alpha i}, d_i \\ s_i s_{\alpha i} & c_i s_{\alpha i} & c_{\alpha i} & c_{\alpha i}, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

- With the shorthands of c = cos, s = sign

5.1 $^0T_1$ ($\alpha_1 = 0$, $a_1 = 0$, $d_1 = 1045$)

Since cos0 = 1 and sin0 = 0:

$$^0T_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 1045 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

5.2 $^1T_2$ ($\alpha_2 = -\pi / 2$, $a_2 = 500$, $d_2 = 0$)

Use cos($-\pi / 2$) = 0 and sin($-\pi / 2$) = -1:

$$^1T_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 500 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

5.3 $^2T_3$ ($\alpha_3 = 0$, $a_3 = 1300$, $d_3 = 0$)

$$^2T_3 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 1300 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

5.4 $^3T_4$ ($\alpha_4 = -\pi / 2$, $a_4 = -55$, $d_4 = 1025$)

Use cos($-\pi / 2$) = 0 and sin($-\pi / 2$) = -1:

$$^3T_4 = \begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & -55 \\ 0 & 0 & 1 & 1025 \\ -\sin(\theta_4) & -\cos(\theta_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

5.5 $^4T_5$ ($\alpha_5 = +\pi / 2$, $a_5 = 0$, $d_5 = 0$)

Use cos($\pi / 2$) = 0 and sin($\pi / 2$) = 1:

$$^4T_5 = \begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

5.6 $^5T_6$ ($\alpha_6 = -\pi / 2$, $a_6 = 0$, $d_6 = 290$)

Use cos($-\pi / 2$) = 0 and sin($-\pi / 2$) = -1:

$$^5T_6 = \begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & 290 \\ -\sin(\theta_6) & -\cos(\theta_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

- Note: theta is the effective angle such that -

$$\theta_1 = -q_1, \quad \theta_2 = q_2, \quad \theta_3 = q_3 - \pi / 2, \quad \theta_4 = -q_4, \quad \theta_5 = q_5, \quad \theta_6 = \pi - q_6.$$

CLOSED-FORM HOMOGENOUS TRANSFORMATION

With rotation from the base to tool frame:

$$R_{06} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}.$$

$$R_{11} = c6,c5,(s1,s4+s_{23},c1,c4)+c6,s5,(c1,c_{23})+s6,(-s1,c4+s_{23},c1,s4),$$
$$R_{21} = c6,c5,(-s1,s_{23},c4+s4,c1)+c6,s5,(-s1,c_{23})+s6,(-s1,s4,s_{23}-c1,c4),$$
$$R_{31} = c6,c5,(c4,c_{23})+c6,s5,(-s_{23})+s6,(s4,c_{23}).$$

$$R_{12} = -s6,c5,(s1,s4+s_{23},c1,c4)-s6,s5,(c1,c_{23})+c6,(-s1,c4+s_{23},c1,s4),$$
$$R_{22} = -s6,c5,(-s1,s_{23},c4+s4,c1)-s6,s5,(-s1,c_{23})+c6,(-s1,s4,s_{23}-c1,c4),$$
$$R_{32} = -s6,c5,(c4,c_{23})-s6,s5,(-s_{23})+c6,(s4,c_{23}).$$

$$R_{13} = -s5,(s1,s4+s_{23},c1,c4)+c5,(c1,c_{23}),$$
$$R_{23} = -s5,(-s1,s_{23},c4+s4,c1)+c5,(-s1,c_{23}),$$
$$R_{33} = -s5,(c4,c_{23})+c5,(-s_{23}).$$

$$p_x = c1,B+290,R_{13},$$
$$p_y = -s1,B+290,R_{23},$$
$$p_z = 1045-1300\sin q_2-1025,s_{23}-55,c_{23}+290,R_{33}.$$

where $B = 500+1300\cos q_2+1025,c_{23}-55,s_{23}.$

INVERSE KINEMATICS

IK DERIVATION PROCESS

Find the wrist center

Solve elbow/arm angles (q2, q3)

Step 2

Step 4

Step 1

Step 3

Solve shoulder base angle q1

Compute wrist rotation and solve wrist angles (q4, q5, q6)
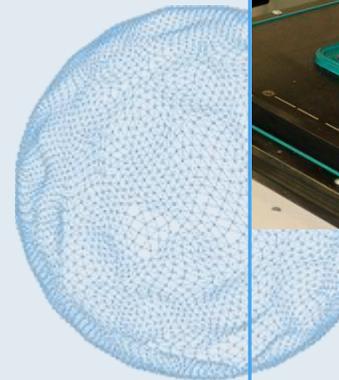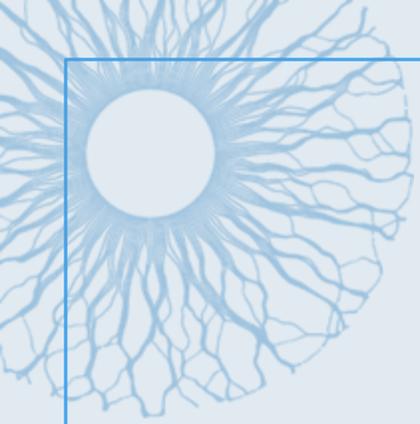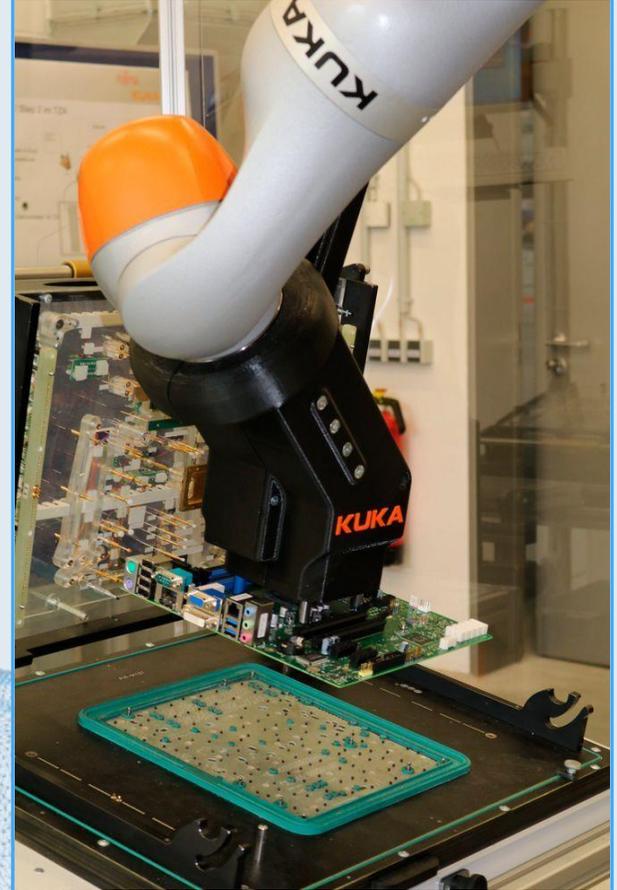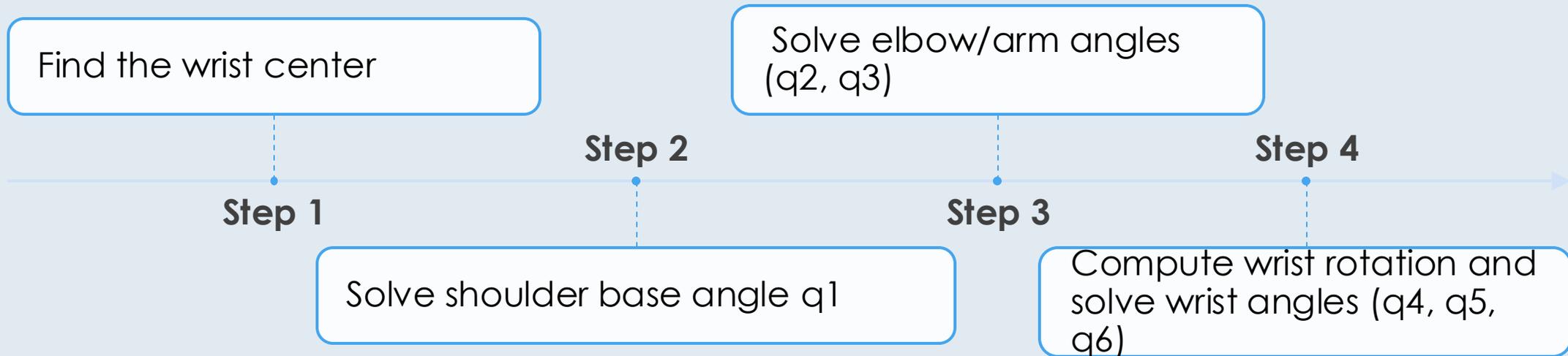
- KUKA KR500 R2830 has a spherical wrist (the last three joint axes intersect), making IK possible.

- Position IK: solve (q1, q2, q3) using the wrist center position.

- Orientation IK: solve (q4, q5, q6) using a reduced wrist rotation.

8 CLOSED–FORM IK SOLUTIONS

In the non-singular case, this robot admits up to 8 closed-form IK solutions. We can enumerate them explicitly as the Cartesian product:

- Shoulder branch $S \in a, b$
- Elbow branch $E \in +, -$
- Wrist branch $W \in 1, 2$

$$q^{(S,E,W)} = \begin{bmatrix} q_1^{(S)} \\ q_2^{(S,E)} \\ q_3^{(S,E)} \\ q_4^{(S,E,W)} \\ q_5^{(S,E,W)} \\ q_6^{(S,E,W)} \end{bmatrix}, \qquad (S,E,W) \in a, b \times +, - \times 1, 2.$$

Wrap the joint limits to reasonable values:

- $q_1 \in [-185, 185]$
- $q_2 \in [-130, 20]$
- $q_3 \in [-100, 144]$
- $q_4 \in [-350, 350]$
- $q_5 \in [-120, 120]$
- $q_6 \in [-350, 350]$

Summary of the solutions:

$$p_{wc} = p_{06} - 290, R_{06} \begin{bmatrix} 0 \backslash 0 \backslash 1 \end{bmatrix} = \begin{bmatrix} x_w \backslash y_w \backslash z_w \end{bmatrix}.$$

$$\rho = \sqrt{x_w^2 + y_w^2},$$

$$q_1^{(a)} = \operatorname{atan2}(-y_w, x_w), \quad B^{(a)} = +\rho, \quad q_1^{(b)} = q_1^{(a)} + \pi, \quad B^{(b)} = -\rho.$$

$$r = B - 500, \quad z = 1045 - z_w,$$

$$L = \sqrt{1025^2 + 55^2}, \quad \phi = \operatorname{atan2}(55, 1025), \quad A = 1300,$$

$$D = \frac{r^2 + z^2 - A^2 - L^2}{2AL}, \quad |D| \le 1,$$

$$\gamma^{(\pm)} = \operatorname{atan2}!\left(\pm\sqrt{1 - D^2}, D\right),$$

$$q_2^{(\pm)} = \operatorname{atan2}(z, r) - \operatorname{atan2}!\left(L\sin\gamma^{(\pm)}, A + L\cos\gamma^{(\pm)}\right), \quad q_3^{(\pm)} = \gamma^{(\pm)} - \phi.$$

$$\theta_5^{(a)} = \operatorname{atan2}(s5, c5), \quad \theta_4^{(a)} = \operatorname{atan2}!\left(R_{36}(3,3), -R_{36}(1,3)\right), \quad \theta_6^{(a)} = \operatorname{atan2}!\left(-R_{36}(2,2), R_{36}(2,1)\right),$$
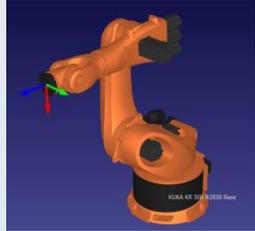
VERIFICATION & VALIDATION

FORWARD KINEMATICS

MATLAB FK Test Case Output:
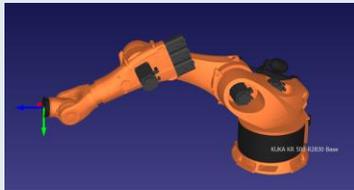
RoboDK FK Test Case Output:

```
[    -0.000000,    -0.000000,     1.000000,  1815.000000 ;
  -0.000000,     1.000000,     0.000000,     0.000000 ;
  -1.000000,    -0.000000,    -0.000000,  2290.000000 ;
   0.000000,     0.000000,     0.000000,     1.000000 ];
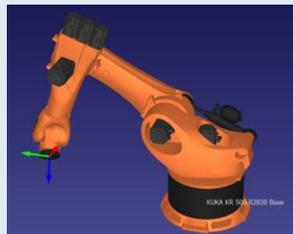```

```
[    -0.000000,    -0.000000,     1.000000,  3115.000000 ;
  -0.000000,     1.000000,     0.000000,     0.000000 ;
  -1.000000,    -0.000000,    -0.000000,   990.000000 ;
   0.000000,     0.000000,     0.000000,     1.000000 ];
```

```
[    -0.342346,     0.288978,     0.894031,  2568.070293 ;
  -0.873978,     0.251354,    -0.415913, -1453.601887 ;
  -0.344908,    -0.923749,     0.166510,  1135.660391 ;
   0.000000,     0.000000,     0.000000,     1.000000 ];
```

```
[    -0.991979,     0.047886,    -0.116978,  1359.464915 ;
  -0.125954,    -0.452114,     0.883022,  1649.464915 ;
  -0.010603,     0.890674,     0.454519,  2919.368529 ;
   0.000000,     0.000000,     0.000000,     1.000000 ];
```

```
[    0.433013,     0.500000,     0.750000,   217.500000 ;
   0.531121,    -0.813798,     0.235889, -1856.312835 ;
   0.728293,     0.296198,    -0.617945,   533.799797 ;
   0.000000,     0.000000,     0.000000,     1.000000 ];
```

```
[    0.767463,    -0.640561,    -0.026114, -1098.824930 ;
   0.170714,     0.243458,    -0.954769,  1613.220607 ;
   0.617945,     0.728293,     0.296198,  1849.528656 ;
   0.000000,     0.000000,     0.000000,     1.000000 ];
```

A	B	C	D	E	F	G	H	I	J	K
q1_deg	q2_deg	q3_deg	q4_deg	q5_deg	q6_deg	Var7	t1	t2	t3	t4
Number	Number	Number	Number	Number	Number	Text	Number	Number	Number	Number
q1_deg	q2_deg	q3_deg	q4_deg	q5_deg	q6_deg		t1	t2	t3	t4
0	-90	90	0	0	0					
							-6.1232e-17	-6.1232e-17	1	1815
							-6.1232e-17	1	6.1232e-17	4.2863e-15
							-1	-6.1232e-17	-6.1232e-17	2290
							0	0	0	1
0	0	0	0	0	0					
							-6.1232e-17	-6.1232e-17	1	3115
							-6.1232e-17	1	6.1232e-17	8.3888e-14
							-1	-6.1232e-17	-6.1232e-17	990
							0	0	0	1
30	-20	40	10	-30	60					
							-0.34235	0.28898	0.89403	2568.0703
							-0.87398	0.25135	-0.41591	-1453.6019
							-0.34491	-0.92375	0.16651	1135.6604
							0	0	0	1
-45	-60	20	-90	45	-30					
							-0.99198	0.047886	-0.11698	1359.4649
							-0.12595	-0.45211	0.88302	1649.4649
							-0.010603	0.89067	0.45452	2919.3685
							0	0	0	1
90	-30	100	120	-60	180					
							0.43301	0.5	0.75	217.5
							0.53112	-0.8138	0.23589	-1856.3128
							0.72829	0.2962	-0.61795	533.7998
							0	0	0	1
-120	10	-80	-150	90	-120					
							0.76746	-0.64056	-0.026114	-1098.8249
							0.17071	0.24346	-0.95477	1613.2206
							0.61795	0.72829	0.2962	1849.5287
							0	0	0	1

INVERSE KINEMATICS

RoboDK IK Test Case Output (a few):
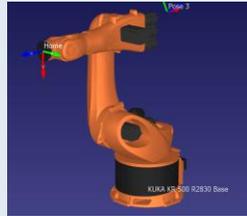
Other configurations (θ1, θ2, θ3, θ4, θ5, θ6)   More options

```
( *)-[   0.00°,  -90.00°,   90.00°,    0.00°,    0.00°,   -0.00°]
( 0)-[   0.00°,  -90.00°,   90.00°,    0.00°,    0.00°,   -0.00°]
( 1)-[   0.00°,  -11.07°,  -96.14°,  180.00°, -107.21°, -180.00°]
( 2)-[   0.00°,  -11.07°,  -96.14°,   -0.00°,  107.21°,    0.00°]
( 3)-[   0.00°,  -11.07°,  -96.14°, -180.00°, -107.21°, -180.00°]
( 4)-[   0.00°,  -11.07°,  -96.14°,  180.00°, -107.21°,  180.00°]
( 5)-[   0.00°,  -11.07°,  -96.14°, -180.00°, -107.21°,  180.00°]
```
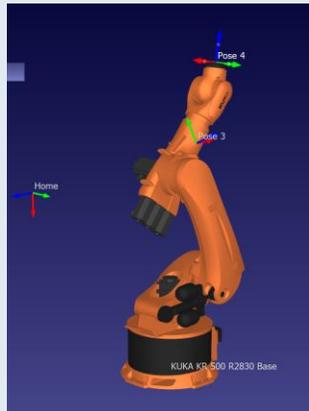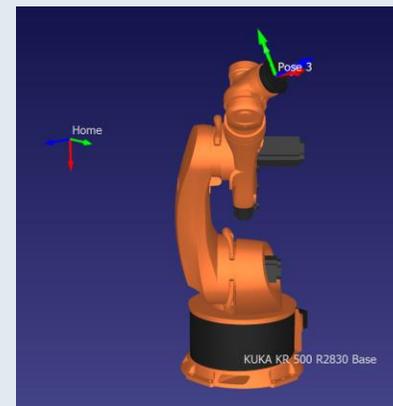
Other configurations (θ1, θ2, θ3, θ4, θ5, θ6)   More options

```
( *)-[-180.00°, -114.07°,   58.86°,   -0.00°,  -25.79°,  180.00°]
( 0)-[-180.00°, -114.07°,   58.86°,   -0.00°,  -25.79°,  180.00°]
( 1)-[ 180.00°,  -60.21°,  -65.00°, -180.00°,  -44.21°,   -0.00°]
( 2)-[ 180.00°, -114.07°,   58.86°,  180.00°,   25.79°,   -0.00°]
( 3)-[-180.00°, -114.07°,   58.86°,   -0.00°,  -25.79°,  180.00°]
( 4)-[  -0.00°, -108.68°,  -26.14°,   -0.00°,   35.82°,   -0.00°]
( 5)-[  -0.00°, -108.68°,  -26.14°,  180.00°,  -35.82°,  180.00°]
( 6)-[  -0.00°, -129.00°,   20.00°,   -0.00°,   10.00°,    0.00°]
( 7)-[  -0.00°, -129.00°,   20.00°,  180.00°,  -10.00°, -180.00°]
( 8)-[-180.00°,  -60.21°,  -65.00°,    0.00°,   44.21°,  180.00°]
( 9)-[-180.00°,  -60.21°,  -65.00°, -180.00°,  -44.21°,   -0.00°]
(10)-[  -0.00°, -114.07°,   58.86°,  180.00°,   25.79°,   -0.00°]
(11)-[ 180.00°,  -60.21°,  -65.00°,    0.00°,   44.21°,  180.00°]
(12)-[ 180.00°,  -60.21°,  -65.00°,  180.00°,  -44.21°,   -0.00°]
(13)-[-180.00°, -114.07°,   58.86°, -180.00°,   25.79°,   -0.00°]
(14)-[  -0.00°, -108.68°,  -26.14°, -180.00°,  -35.82°,  180.00°]
(15)-[  -0.00°, -129.00°,   20.00°, -180.00°,  -10.00°,  180.00°]
(16)-[-180.00°,  -60.21°,  -65.00°,  180.00°,  -44.21°,   -0.00°]
(17)-[-180.00°, -114.07°,   58.86°, -180.00°,   25.79°,   -0.00°]
(18)-[ 180.00°,  -60.21°,  -65.00°,    0.00°,   44.21°, -180.00°]
(19)-[ 180.00°, -114.07°,   58.86°,   -0.00°,  -25.79°, -180.00°]
(20)-[  -0.00°, -108.68°,  -26.14°,  180.00°,  -35.82°, -180.00°]
(21)-[  -0.00°, -129.00°,   20.00°,  180.00°,  -10.00°,  180.00°]
(22)-[-180.00°,  -60.21°,  -65.00°,    0.00°,   44.21°, -180.00°]
(23)-[-180.00°, -114.07°,   58.86°,   -0.00°,  -25.79°, -180.00°]
(24)-[  -0.00°, -108.68°,  -26.14°, -180.00°,  -35.82°, -180.00°]
(25)-[  -0.00°, -129.00°,   20.00°, -180.00°,  -10.00°,  180.00°]
```

Tested 18 angle combinations, all T values were same as FK

Note: RoboDK has more configurations than MATLAB (8) because it does not have angle limits. I imposed angle limits on MATLAB, so that it would only give out feasible solutions.

MATLAB IK Test Case Output:

	test_id	q1_deg	q2_deg	q3_deg	q4_deg	q5_deg	q6_deg	n_solutions	max_pos_err_mm	max_ang_err_deg	recovered_original	n_wrist_singular_flags
1												
2	1	0	-90	90	0	0	0	3	2.41019985e-13	0	1	1
3	2	0	0	0	0	0	0	3	5.684341886e-13	0	1	1
4	3	30	-20	40	10	-30	60	4	8.198074455e-13	0	1	0
5	4	-45	-60	20	-90	45	-30	4	6.431098711e-13	1.20741827e-06	1	0
6	5	20	-40	60	90	0.01	45	4	8.276526716e-13	0	1	0
7	6	-60	-30	80	-120	-0.01	-90	4	6.431098711e-13	0	1	0
8	7	184	-60	40	0	10	0	4	1.113898972e-12	0	1	0
9	8	-184	-60	40	0	10	0	4	1.888074598e-12	1.20741827e-06	1	0
10	9	0	19	20	0	10	0	4	6.108434619e-14	0	1	0
11	10	0	-129	20	0	10	0	8	1.016868106e-12	0	1	0
12	11	0	-60	143	0	10	0	2	1.049094726e-12	0	1	0
13	12	0	-60	-99	0	10	0	2	9.112940061e-13	0	1	0
14	13	0	-60	40	349	10	0	4	4.556346636e-13	1.707547293e-06	1	0
15	14	0	-60	40	-349	10	0	4	4.592217417e-13	1.707547293e-06	1	0
16	15	0	-60	40	0	119	0	4	6.485459386e-13	0	1	0
17	16	0	-60	40	0	-119	0	4	8.198074455e-13	1.20741827e-06	1	0
18	17	0	-60	40	0	10	349	4	6.448353174e-13	1.20741827e-06	1	0
19	18	0	-60	40	0	10	-349	4	6.448353174e-13	1.20741827e-06	1	0

Other configurations (θ1, θ2, θ3, θ4, θ5, θ6)   More options

```
( *)-[ -45.00°,  -60.00°,   20.00°,   90.00°,  -45.00°,  150.00°]
( 0)-[ -45.00°,  -60.00°,   20.00°,   90.00°,  -45.00°,  150.00°]
( 1)-[ -45.00°,  -39.68°,  -26.14°,  113.54°,  -50.47°,  115.62°]
( 2)-[ -45.00°,  -60.00°,   20.00°,  -90.00°,   45.00°,  -30.00°]
( 3)-[ -45.00°,  -39.68°,  -26.14°,  -66.46°,   50.47°,  -64.38°]
( 4)-[ -45.00°,  -39.68°,  -26.14°,  293.54°,   50.47°,  -64.38°]
( 5)-[ -45.00°,  -39.68°,  -26.14°, -246.46°,  -50.47°,  115.62°]
( 6)-[ -45.00°,  -60.00°,   20.00°,  270.00°,   45.00°,  -30.00°]
( 7)-[ -45.00°,  -60.00°,   20.00°, -270.00°,  -45.00°,  150.00°]
( 8)-[ -45.00°,  -39.68°,  -26.14°,  -66.46°,   50.47°,  295.62°]
( 9)-[ -45.00°,  -39.68°,  -26.14°,  113.54°,  -50.47°, -244.38°]
(10)-[ -45.00°,  -60.00°,   20.00°,  -90.00°,   45.00°,  330.00°]
(11)-[ -45.00°,  -60.00°,   20.00°,   90.00°,  -45.00°, -210.00°]
(12)-[ -45.00°,  -39.68°,  -26.14°,  293.54°,   50.47°,  295.62°]
(13)-[ -45.00°,  -39.68°,  -26.14°, -246.46°,  -50.47°, -244.38°]
(14)-[ -45.00°,  -60.00°,   20.00°,  270.00°,   45.00°,  330.00°]
(15)-[ -45.00°,  -60.00°,   20.00°, -270.00°,  -45.00°, -210.00°]
```

MOTION PLANNING APP LIVE DEMO

APP START COMMANDS

Github: https://github.com/maxheil5/ME7751_RideRo
bot_Project/tree/main

Start Backend (Terminal 1):

```
cd "c:\Users\maxhe\OneDrive\Documents\Github\ME7751_RideRobot_Project\4 - Simulation Environment\server"
python -m ensure pip --upgrade
python -m pip install --upgrade pip
python -m pip install numpy
npm install
npm run dev
```

Start Frontend (Terminal 2):

```
cd "c:\Users\maxhe\OneDrive\Documents\Github\ME7751_RideRobot_Project\4 - Simulation Environment\app"
npm install
npm run dev
```

Open http://localhost:5173.

ME7751 Task D: Interactive Motion Planner

MoveJ and MoveL planning using backend Python FK/IK wrappers integrated with your existing modified DH parameters.

Waypoints

[Joint Mode] [Pose Mode]

[Set Start to Current]   [Set Goal to Current]

Start

q1 (deg)	q2 (deg)	q3 (deg)
0	-90	90

q4 (deg)	q5 (deg)	q6 (deg)
0	0	0

Goal

q1 (deg)	q2 (deg)	q3 (deg)
30	-20	40

q4 (deg)	q5 (deg)	q6 (deg)
10	-30	60

Planner Controls

Motion Type
[MoveJ ▾]

MoveJ Profile
[cubic ▾]

Steps (100)
[———————●————]

[Plan] [Play] [Pause] [Reset]

Sanity Checks

[1) FK on [0,-90,90,0,0,0]]

[2) IK round-trip on Test 1 pose]

[3) MoveJ home -> moderate pose]

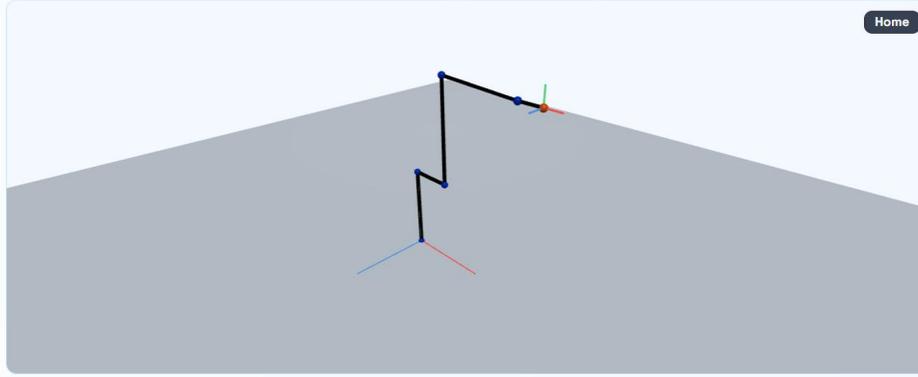[4) MoveL between random reachable poses]

Status
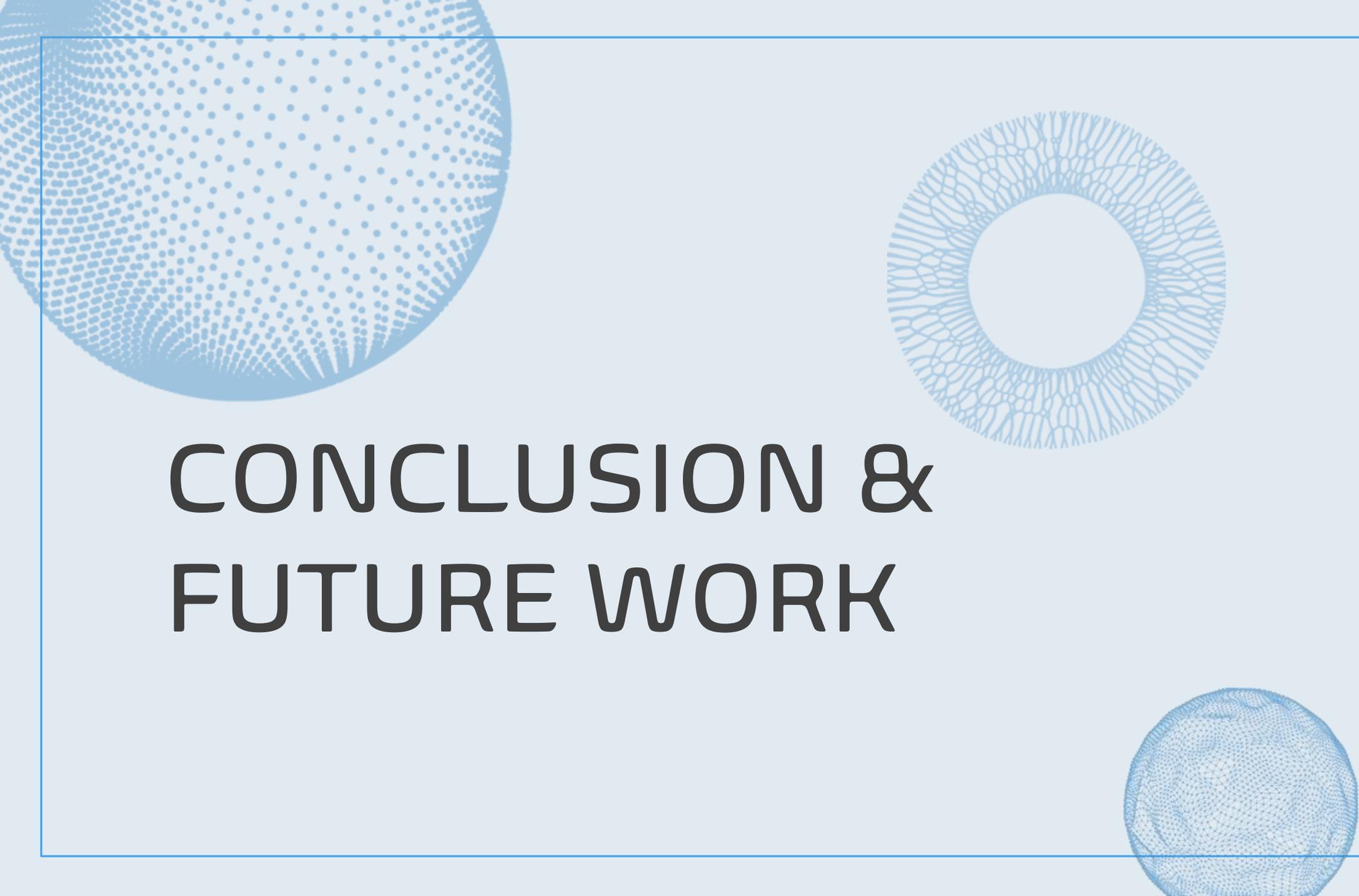
No warnings or errors.

Robot Viewer

Three.js (hybrid: URDF if available, procedural chain always)

```
q = [0.0, -90.0, 90.0, 0.0, 0.0, 0.0]
TCP: x=1815.0 mm, y=0.0 mm, z=2290.0 mm | RPY=0.0, 90.0, 0.0 deg
URDF state: not configured
```

[Home]

Joint Trajectory

Plan a trajectory to view q1..q6 plots.

# CONCLUSION & FUTURE WORK

FUTURE INITIATIVES

Make more robust simulation package. Find/create a .urdf file for KUKA KR500 R2830 robot geometry.

Incorporate mobile robot (bottom half). Investigate the effect of the mobile robot portion of the system and revaluate FK and IK.

Build out scaled model with 3D-printing. Wire a scaled model of the ride system and execute planned motion paths to simulate similar motion during the ride.

Challenges & Lessons Learned:

- Inverse Kinematics starting point

  o Hard to decide/figure out where the first place to begin

- Achieving robust simulations in a clean virtual environment

- Finding resources on KUKA Robots (paywalls)
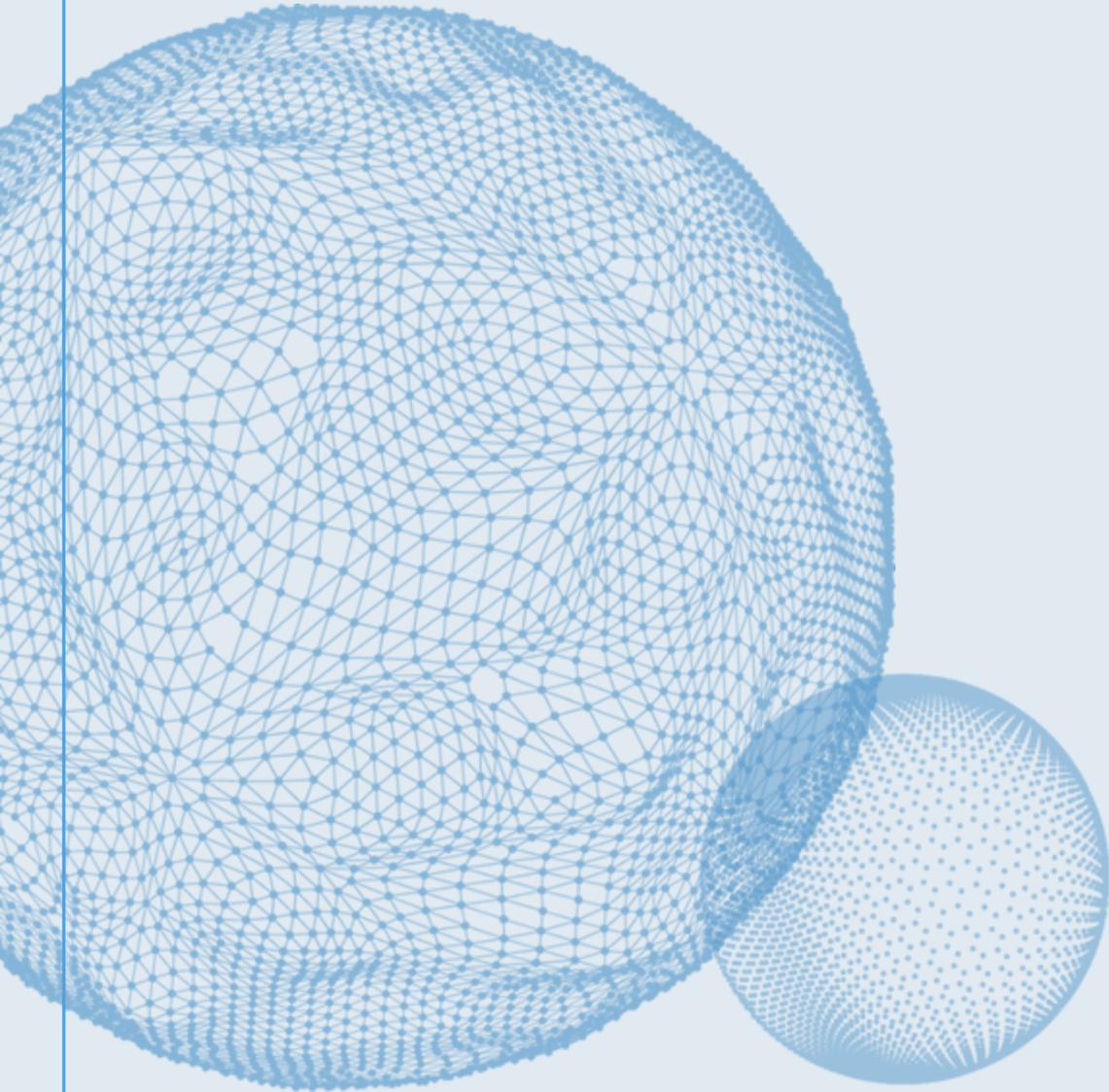
THANK YOU

MAX HEIL

HEIL.115@OSU.EDU

HTTPS://GITHUB.COM/MAXHEIL5/ME77
51_RIDEROBOT_PROJECT/TREE/MAIN